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An Experimental Study of Diffusion From a Line Source in a Turbulent Boundary Layer

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This paper summarizes part of an experimental investigation (Collins, 1973) concerning slot injection of Poly (ethylene oxide) solution concentrate into a two-dimentional developing turbulent boundary layer in channel flow. Experimentally measured velocity and concentration profiles were used to evaluate the effect of injection concentration and rate on the concentration boundary layers. These profiles were used to evaluate wall concentration and diffusing plume thickness for injections of pure water and various concentrations of Poly(ethylene oxide). The data are the only experimental results for turbulent diffusion measurements close to the source.

DIFFUSION IN A TURBULENT BOUNDARY LAYER

An experimental study of diffusion of ammonia from a slot into a turbulent air boundary layer serves as the basis for characterizing turbulent diffusion. Poreh and Cermak (1964) found a series of four stages or zones in the development of a concentration boundary layer. They defined a relative rate parameter β to differentiate between zones:

$$L_\delta = \delta \Big/ \frac{d\delta}{dx} \tag{1}$$

$$L_\lambda = \lambda \Big/ \frac{d\lambda}{dx} \tag{2}$$

$$\beta = L_\lambda / L_\delta \tag{3}$$

Initial Zone: In this region closest to the source, the concentration boundary layer grows slowly by molecular diffusion through the laminar sublayer. The characteristic length λ is the same order of magnitude as the laminar sublayer.

Intermediate Zone: In this zone, there is very rapid growth due to high turbulence intensity near the wall. The diffusing plume is submerged in the boundary layer and is considerably larger than the laminar sublayer. Axial concentration gradients are much smaller than vertical gradients, and the ratio β is small. The mean concentration profiles can be described by the following dimensionless curve, proposed by Morkovin (1965):

$$\frac{C}{C_w} = \exp\left[-0.693(y/\lambda)^{1.5}\right] \tag{4}$$

The intermediate zone lies in the range $0.15 < \lambda/\delta < 0.36$ and $0.08 < \beta < 0.38$.

Transition Zone: In this region, the plume growth rate is slowed because of less turbulence in the outer region of the boundary layer. The profile begins to gradually change shape. This region begins approximately 18 boundary-layer thickness from the source and ends at 60. At the end of this zone β increases to unity, and the value of λ/δ approaches 0.64.

The Final Zone: In the final zone, the concentration boundary layer coincides with the shear layer. The value of λ/δ remains constant (0.64) and is expressed as

$$\frac{C}{C_w} = \exp\left[-0.693(y/\lambda)^{2.15}\right] \tag{5}$$

EXPERIMENT

The investigation was conducted in the water tunnel of the School of Chemical Engineering, Georgia Institute of Technology. The apparatus was a single-pass, continuous flow, low-speed water tunnel consisting of an inlet section, a test section, and an exit duct, all constructed of 8 mm aluminum plate. The system was operated at the maximum capacity of 900 l of water/min for all runs. This gave a free stream velocity and boundary-layer thickness of 0.8 m/s and 2 cm, respectively.

Concentration measurements were made on 25 ml samples, which were withdrawn through a Pitot probe. Analyses were determined colormetrically (Bausch and Lomb Spectronic 20) from the intensity of a tracer

Table 1. Summary of Wall Concentration Results

Injected concentration C_ip.p.m	Injection rate cm^3/min	Wall concentration $(C_w/C_i) \times 10^3$ Station 1	Station 2	Station 3	Station 4
0	250	15.5	3.4	2.2	1.5
		14.5	3.5		
	350	25.0	6.1	4.4	3.1
		22.0	6.8		
	500	37.0	10.2	6.6	5.0
		37.0	11.8		
	1 000	68.0	18.0	10.5	8.4
		55.0	18.8		
500	250	40.0	15.0	7.2	2.5
	350	66.0	22.5	12.0	3.9
	500	90.0	33.0	15.0	5.7
	1 000	140.0	56.0	24.0	9.6
1 000	250	76.0	27.5	16.4	6.2
	350	115.0	46.0	29.0	10.2
	500	200.0	66.0	36.0	12.0
	1 000	250.0	115.0	60.0	17.0
2 000	250	33.0	20.0	11.5	4.5
		22.0	16.0	10.5	5.4
	350	37.0	32.0	18.0	8.5
		58.0	26.0	21.0	9.5
	500	180.0	160.0	58.0	38.0
				29.0	32.0
	1 000	400.0	400.0	105.0	62.0
				48.0	38.0

Table 2. Relative Rate Parameter and Ratio of Plume Height to Boundary-Layer Thickness

Distance from channel inlet (cm)	Injection concentration p.p.m.	β	$\frac{\lambda_{avg}}{\delta}$
122	0	0.033	0.079
137	0	0.114	0.276
152	0	0.390	0.388
167	0	0.370	0.434
122	500	0.082	0.034
137	500	0.077	0.082
152	500	0.064	0.174
167	500	0.065	0.428
122	1 000	0.141	0.032
137	1 000	0.096	0.046
152	1 000	0.069	0.066
167	1 000	0.117	0.228
122	2 000	0.186	0.024
137	2 000	0.142	0.033
152	2 000	0.138	0.059
167	2 000	0.133	0.100

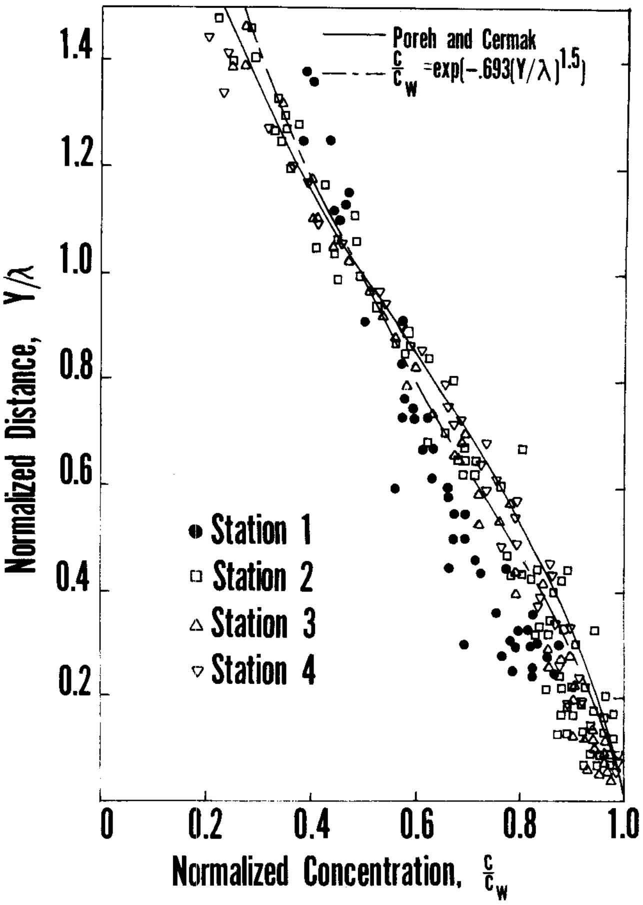

Fig. 1. Normalized concentration profile injected solution—water.

(Rhodamine B dye), which was added to the injected concentrate. Concentration profiles were obtained at stations located 8, 23, 38, and 53 cm downstream from the injection source.

Runs were made without mass addition, with water addition, and with polymer injections at various injection rates and concentrations. Polymer solutions of 500, 1 000, and 2 000 ppm were injected into the boundary layer at rates of 250, 350, 500, and 1 000 cm^3/min. The injection rates were selected so that the maximum corresponded to the sublayer discharge at 20°C. The sublayer discharge, a term first used by Wu (1972), is defined as the flow rate through the sublayer, with no polymer addition.

RESULTS

Wall concentration was determined by extrapolating a plot of log (C/C_i) vs. y to zero. Least-squares fitting was discarded because of insufficient amounts of data and irregularity of the concentration profile. After the wall concentration was found, λ was determined from the plot. The values of C_w/C_i and λ were used as a basis for comparing injection rates and concentrations. The relative rate parameter β was determined from the rate of growth of both the concentration and momentum boundary layers. A plot of λ vs. x and least-squares fits of λ and δ vs. x were used to evaluate β, which was used to compare results with other investigators, and to determine the zones of concentration boundary-layer development. Tables 1 and 2 summarize the results for λ, β, and C_w.

The results in Table 1 show that wall concentration increased with injection rate and decreased with distance from the source. Both these results are to be expected. The normalized wall concentration (C_w/C_i) increased with increasing C_i for a given injection rate and station, except for some of the 2 000 ppm cases. This indicates that turbulent diffusion is suppressed. [Dye patterns for 2 000 ppm injections showed wavering streaks near the source; this was not as pronounced for other concentrations. Perhaps these streaks caused errors in the 2 000 ppm determinations giving some low results. Wetzel and Ripkin (1970) also noticed streaking of PEO solutions near the slot.] Another result of reduced turbulent diffusion is decreased growth rate of the concentration boundary layer. Suppressed diffusion was also noticed by other investigators (Wu, 1972).

Concentration data for pure water injection are shown in Figure 1 and are compared with curves of Morkovin (1965) and Poreh and Cermak (1964) for the intermediate zone of diffusion. Based on the criteria of Poreh and Cermak, stations 2, 3, and perhaps station 4 should be in the intermediate zone, and station 1 is in the initial zone. Although there is some scatter in the data, the

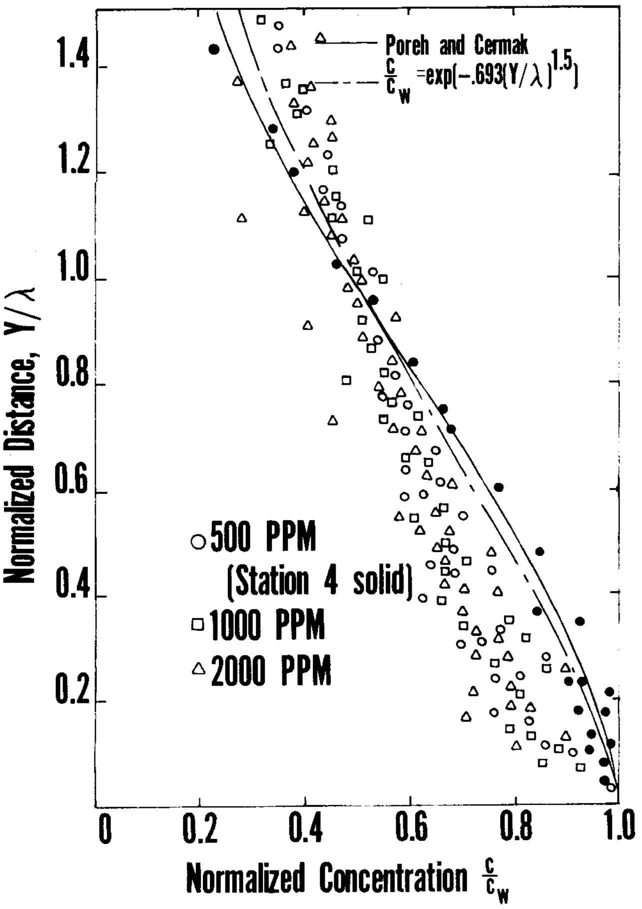

Fig. 2. Normalized concentration profile injected solution—PEO.

results show good agreement with the suggested curves for the intermediate zone at stations 2, 3, and 4. At station 1 there appears to be considerable deviation from the suggested curve. Since station 1 is in the initial zone, this deviation is expected.

The concentration profiles for injection of polymer solutions are shown in Figure 2. Only the data at station 4 for 500 ppm injection appear to be in the intermediate zone. The other data are to the left of the curves, just like station 1—water injection. For the 500 ppm injection, the concentration layer thickness at station 4 has almost reached that of the water layer. The relative rate parameter β indicates that station 4 should be in the initial zone. However, because the λ/δ ratio indicates that the diffusion has reached the intermediate zone, this is a reasonable result.

All the data for polymer injection (except as noted) deviate considerably from the intermediate zone curves and appear to be in the initial zone. The relative rate parameter β (for 1 000 and 2 000 ppm) indicates that the diffusion process should be in the intermediate zone. However, the ratio λ/δ (except for station 4 of the 1 000 ppm injection) indicates that diffusion should be in the initial zone.

Overall, the data indicate that the initial zone has been stretched out, and the growth rate of the concentration boundary layer λ is slowed by increased polymer concentration. The relative rate parameter β alone cannot characterize diffusion in the initial zone with polymer addition. PEO retards the rate of diffusion, hence, $d\lambda/dx$ decreases, and L_λ and β increase. However, normalized concentration profiles indicate the initial zone or turbulent diffusion.

NOTATION

C = polymer concentration (ppm)
C_i = polymer concentration (ppm) of injected solution
C_w = polymer concentration (ppm) at the wall
L_δ = boundary-layer growth parameter defined by Equation (1)
L_λ = concentration plume growth parameter defined by Equation (2)
x = coordinate parallel to the test surface
y = coordinate normal to the test surface
β = relative rate parameter, defined in Equation (3)
δ = boundary-layer thickness $\delta = y(0.99\ U)$
ξ = nondimensional distance — $\xi = y/\lambda$
λ = characteristic concentration plume height — $y(0.5\ C_w)$

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A Model for the Residence Time Distribution of Liquid Phase in Trickle Beds

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The residence time distribution of the liquid phase has been covered by Satterfield (1975) in his excellent review on trickle-bed reactors. The earlier models included the dispersed plug flow model (Otake and Kunigita, 1958; Sater and Levenspiel, 1966; Furzer and Michell, 1970; Pham Co and Ribaud, 1971), the plug flow-stagnancy model (Hoogendoorn and Lips, 1965), the dispersed plug flow-stagnancy model (van Swaaij et al., 1969; Bennett and Goodridge, 1970), the time-delay model (Buffham et al., 1970), and the laminar film—partial mixer model (Michell and Furzer, 1972). A comparison of the models was given by Schwartz and Roberts (1973). The Michell